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Characterization of Nash Equilibrium Strategy of a Two Person Non Zero Sum Games with Trapezoidal Fuzzy Payoffs

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Abstract: In this article, we have considered the Bimatrix game with symmetric trapezoidal Pay-off's and then we define Nash Equilibrium solution for pure and mixed strategies. We extended the models of investigating Nash Equilibrium strategy of a two person Non zero sum games with fuzzy payoff's in trapezoidal fuzzy environment and also proposed the methodology for the existence of equilibrium strategies. In addition to that, numerical examples are presented to find Nash Equilibrium for the Bimatrix game.

Keywords: Fuzzy numbers, Bimatrix games, symmetric Bimatrix games, Nash Equilibrium, Trapezoidal Fuzzy number.

INTRODUCTION

Game theory is a mathematically founded study of conflict and co-operation. Game theoretic concept apply when the decisions / actions of independent agents affects the interest of others.

In 1944, Von Neumann & Morgenstern [8] introduced game theory in their pioneer work "Theory of Games & Economic Behaviour".

Gamer are broadly classified in to two type's co-operative & non co-operative games. Two person non zero sum games which is also known as Bimatrix games comes under the category of non-co-operative game. Basically in real life games, players are not able to estimate exact payoffs due to imprecision of the available information. In 1965, Zadeh [16] introduced the concept of fuzzy set theory to overcome there kind of uncertainty.

The most commonly used solution concept in traditional game theory is that of Nash Equilibrium which has been introduced by John Nash.

The extension of Nash Equilibrium concept for two person non zero sum games with fuzzy payoffs was introduced by Maeda [5] fuzziness, in Bimatrix games, were studied by many authors.

Nayak & Pal [7] described Bimatrix games with interval pay-off & its Nash Equilibrium strategy. Therefore, Inthis paper, we have considered Bimatrix game with trapezoidal fuzzy payoff. Also we defined Nash Equilibrium solution of such games & tried to get the solution of a trapezoidal fuzzy number. The numerical examples illustrate the theory.

PRELIMINARIES

Bimatrix Game [11]

A two person finite game in a strategic form which is defined as the matrix of ordered pairs is called a Bimatrix game. A **Bimatrix game** is a 2 player regular game where

Player 1 with a finite set of strategy $S = \{s_1, s_2, \ldots\ldots s_m\}$

Player 2 with a finite set of strategy$T = \{t_1, t_2, \ldots\ldots t_n\}$

When the pair of strategies (s_i, t_j) is choosen, the first player's payoff is $a_{ij} = u_1(s_i, t_j)$ and the second player's payoff is $b_{ij} = u_2(s_i, t_j)$ such that u_1, u_2 are called the **payoff functions.**

The outcomes of payoff values can be represented by a Bimatrix.

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Player 2

	Strategy	t_1	t_2	...	t_n
	s_1	(a_{11}, b_{11})	(a_{12}, b_{12})	...	(a_{1n}, b_{1n})
Player 1	s_2	(a_{21}, b_{21})	(a_{22}, b_{22})	...	(a_{2n}, b_{2n})
	...	...	...	...	...
	s_m	(a_{m1}, b_{m1})	(a_{m2}, b_{m2})	...	(a_{mn}, b_{mn})

The payoff matrix of Player 1 and Player 2 is

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}, \; B = \begin{pmatrix} b_{11} & b_{12} \cdots & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & b_{mn} \end{pmatrix}$$

Player 1's payoff representation is the first component of the ordered pairs and the Player 2's payoff representation is the second component of the ordered pairs

Symmetric Bimatrix Games [13]

A 2– Player Strategic game is symmetric, if the player's sets of Pure Strategies are the Same and The Player's Payoff functions u_1 and $u_2$are such that $U_1(S_1,S_2) = U_2(S_2,S_1)$

(i.e.) a symmetric game does not change when the player change roles.

Using the notation of Bimatrix games, an m x n Bimatrix game Γ = (A,B) is symmetric if

1. m = n and
2. $a_{ij} = b_{ji}$ for all j ∈ {1,2,.........n}

Or equivalently B $= A^T$

Equilibrium Point

In a Bimatrix game, Player1 has finite set of strategies $S = \{s_1, s_2, \ldots.. s_m\}$and Player 2 has finite set of strategies $T = \{t_1, t_2, \ldots.. t_n\}$ then,

The pair of strategies (s^*, t^*) is called an **equilibrium point** if

i. $u_1(s,t^*) \leq u_1(s^*, t^*)$ for all $s \in S$and
ii. $u_2(s^*,t^*) \leq u_2(s^*, t^*)$For all $t \in T$

We can easily verify that if $(s^*, t^*) = (s_i, t_j)$is an equilibrium point then

a_{ij} isthe maximum in the column j of the matrix $A = a_{ij} = max\, a_{kj},\ 1 \leq k \leq m$

b_{ij} is the maximum in the row i of the matrix $B = b_{ij} = \max b_{ik},\ 1 \leq k \leq n$

Nash Equilibrium [4]

A Nash equilibrium also called strategic equilibrium, is a list of strategies one for each player, which has the property that no player can unilaterally change his strategy to get a better payoff. A Nash equilibrium for a game $\urcorner = (\tilde{x}, \tilde{y})$ is a Nash equilibrium for a Bimatrix game $\urcorner = (A, B)$ if

(i) For every mixed strategy x of the row player $x^T Ay \leq \tilde{x}^T A\tilde{y}$and
(ii) For every mixed strategy y of the column player$x^T By \leq \tilde{x}^T B\tilde{y}$

Expected Payoff

For a mixed strategy Nash equilibrium the **expected payoff** for that player is given by multiplying each probability in each cell by his/her respective payoff in that cell.

Therefore, The Expected payoffs are defined by the relations

Player 1: $\pi_1(p, q) = \Sigma\Sigma p_i q_j a_{ij}$ where $i = 1\, tom$ and $j = 1ton$

Player 2: $\pi_2(p,q) = \Sigma\Sigma p_i q_j b_{ij}$ where $i = 1tom$ and $j = 1ton$

FUZZY SET

A fuzzy set Ã is defined by $\mu_{\tilde{A}}(x)$: R $\mu \rightarrow [0, 1]$Ã = $\{(x, \mu_A(x)): x \in A,\ \mu_A(x) \ \in [0,1]\}$

In the pair$(x, \mu_A(x))$, the first element x belong to the classical set A, the second element$\mu_A(x)$, belong to the interval $[0,1]$ called Membership function.

Fuzzy Number

A fuzzy subset $\tilde{A}$ defined on R, is said to be a fuzzy number if its membership function $\mu_{\tilde{A}}$(x) satisfies the following conditions

- There existat least one $x_0 \in R$, $\mu_{\tilde{A}}(x_0)$ =1
- $\mu_{\tilde{A}}$(x) is piecewise continuous
- $\tilde{A}$ must be normal and convex

Triangular Fuzzy Number

A fuzzy number $\tilde{A}$ = (a_1, a_2, a_3) is said to be triangular fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for x < a_1 \\ \dfrac{(x - a_1)}{(a_2 - a_1)} & for a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{(a_3 - a_2)} & for a_2 \le x \le a_3 \\ 0 & for x > a_3 \end{cases}$$

where$a_1 \le a_2 \le a_3$are real numbers.

Trapezoidal Fuzzy Number [16]

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number if its membership function is given by,

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & x > a_4 \end{cases}$$

where$a_1 \le a_2 \le a_3 \le a_4$are real numbers.

Defuzzification of Trapezoidal Fuzzy Number

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ is agiven trapezoidal fuzzy number. Then the defuzzification of the fuzzy number by graded mean integration method is

$$P_1\,(c) = \frac{c_1 + 2c_2 + 2c_3 + c_4}{6}$$

ILLUSTRATIONS

Numerical example: 1

Consider a trapezoidal fuzzy bimatrix and computing the equilibrium solutions strategies.

		Player B	
		B_1	B_2
Player A	A_1	(T (0.3, 0.5, 0.8,1.0), (0.1, 0.4,0.9,1.0))	(T (0.3, 0.5, 0.6,1.0), (0.1, 0.4,0.9,1.0))
	A_2	(T (0.4, 0.8, 0.8,1.0), (0.4, 0.4,1.0,1.1))	(T (0.4, 0.5, 0.8,1.0), (0.4, 0.4,1.0,1.1))
	A_3	(T (0.3, 0.7, 0.8,1.0), (0.1, 0.4,0.8,1.0))	(T (0.3, 0.7, 0.7,1.0), (0.1, 0.4,0.8,1.0))

Solution:

After Defuzzification

	Player B	
	B_1	B_2
A_1	(0.65, 0.616)	(0.583, 0.616)
A_2	(0.76, 0.716)	(0.66, 0.716)
Player A A_3	(0.716, 0.583)	(0.683, 0.583)

Where payoff matrix A = $\begin{pmatrix} 0.65 & 0.583 \\ 0.76 & 0.66 \\ 0.716 & 0.683 \end{pmatrix}$

Where payoff matrix B = $\begin{pmatrix} 0.616 & 0.616 \\ 0.716 & 0.716 \\ 0.583 & 0.583 \end{pmatrix}$

By best reply response method,

Best reply of Player 1 to the strategic T of Player 2 is defined as the set

$$R_1(t) = \{s^* \in S;\ u_1(s^*, t) \geq u_1(s, t), \forall\ s \epsilon S$$

Similarly Best reply of Player 2 to the strategy S of Player 1 is defined as

$$R_2\ (S) = \{t^* \in T;\ u_2(s, t^*) \geq\ u_2(s, t), \forall\ t \epsilon T\}$$

Therefore (A_1, B1) => Not a Pure Nash Equilibrium

Since (0.76 > 0.65)

Therefore (A_1, B2) => Not a Pure Nash Equilibrium

Since (0.66 > 0.583)

After verifying all the set of payoff's

(A_2, B_1) and (A_3, B_2) are the two Pure Nash Equilibrium points.

Example 2:

The below are the scores of Team A and Team B in a cricket tournament following the strategies to win the tournament. Computing the Equilibrium solution strategies for the given trapezoidal fuzzy bimatrix game.

		Team B	
		B_1	B_2
Team A	A_1	T((50,60,70,80),80)	T((30,40,50,60),40)
	A_2	T((20,30,40,50),50)	T((50,60,70,80),10)

Solution:

After Defuzzification

		Team B		
		B_1	B_2	
Team A	A_1	T(65,80)	T(45,40)	P_1
	A_2	T(35,50)	T(65,10)	P_2
		q_1	q_2	

Expected payoff of A_1 and A_2 given by

E (A_1) = $65q_1 + 45q_2$

E (A_2) = $35q_1 + 65q_2$

Since E (A_1) = E (A_2) therefore we have,

$q_2 = {}^3/_5$

$q_1 = {}^{2}/_{5}$

$\left({}^{2}/_{5}, {}^{3}/_{5}\right)$ => (0.4, 0.6) is the mixed Nash Equilibrium

Similarly E (B_1) = E (B_2) yields another Mixed Nash Equilibrium.

Example 3:

Let $\tilde{A}$ be a symmetric trapezoidal fuzzy payoff matrix given by

$$\tilde{A} = \begin{pmatrix} (8,12,20,30) & (1,4,5,8) \\ (4,5,9,20) & (8,10,12,30) \end{pmatrix}$$

After Defuzzification,

$$\tilde{A} = \begin{pmatrix} 17 & 4.5 \\ 8.66 & 13.66 \end{pmatrix}$$

Since it is a symmetric bimatrix, we have $\tilde{A}=B^T$

$$B^T = \begin{pmatrix} 17 & 4.5 \\ 8.66 & 13.66 \end{pmatrix}$$

Therefore B = $\begin{pmatrix} 17 & 8.66 \\ 4.5 & 13.60 \end{pmatrix}$

Since $\tilde{A}=B^T$, therefore the bimatrix is given by

		Player B	
		B_1	B_2
Player A	A_1	(17,17)	(4.5,8.66)
	A_2	(8.66,4.5)	(13.66,13.66)

By dominance strategy method,

The strategy s_k ∈S of the Player 1 is called dominating, another strategy s_i ∈S for each strategy t ϵ T of the Player 2 we have $U_1\ (s_k\ , t)\ \geq\ U_1\ (s_i, t)$

Dominating strategy of the Player 2 defined in the same way

(A_1, B_1) and (A_2, B_2) are the two pure dominating strategy Nash equilibrium points

Example 4:

Consider the below fuzzy bimatrix game and analysing the equilibrium solution strategies

		Player B		
		B_1	B_2	B_3
Player A	A_1	(T (4,5,6,7),(1,3,5,6))	(T (5,6,7,8),(2,3,4,5))	(T (4,5,6,7),(1,3,4,5))
	A_2	(T (2,3,4,5),(0,1,2,3))	(T (1,3,5,6),(3,4,5,7))	(T (2,3,4,5),(1,3,5,7))
	A_3	(T (3,4,5,6),(2,4,6,8))	(T(3,5,7,8),(1,3,5,6))	(T(6,7,8,9),(4,6,7,8))

Solution:

After Defuzzification

		Player B			
		B_1	B_2	B_3	
Player A	A_1	(5.5,3.8)	(6.5,3.5)	(5.5,3.3)	P_1
	A_2	(3.5,1.5)	(3.8,4.6)	(3.5,4)	P_2
	A_3	(4.5,5)	(5.8,3.8)	(7.5,6.3)	P_3
		q_1	q_2	$1-q_1-q_2$	

By expected payoff method,

E (B_1) = E (B_2) = E (B_3)

Since, E (B_1) = E (B_2)

We have 0.9 P_1 + 4.3 P_2 = 1.2

Similarly, E (B_2) = E (B_3)

Therefore we have 2.7 P_1 + 3 P_2 = 2.5

Solving the equations, we have P_1=0.802 and P_2 = 0.198

Therefore (0.802, 0.198) are the two mixed Nash Equilibrium points for the given trapezoidal bimatrix game.

CONCLUSION

In this paper, we have considered a bimatrix game whose payoff elements are symmetric trapezoidal fuzzy numbers. We generalise the existence conditions for all types of Nash Equilibrium strategies. Some of the numerical examples establish the theory on strong ground. Therefore we proposed a methodology for formalising and constructing equlibria in Fuzzy Bimatrix games.

REFERENCES

[1] Dutta, B., & Gupta, S. K. (2014). On Nash equilibrium strategy of two-person zero-sum games with trapezoidal fuzzy payoffs. *Fuzzy Information and Engineering*, *6*(3), 299-314.

[2] Cunlin, L., & Qiang, Z. (2011). Nash equilibrium strategy for fuzzy non-cooperative games. *Fuzzy Sets and Systems*, *176*(1), 46-55.

[3] Qiu, D., Xing, Y., & Chen, S. (2018). Solving fuzzy matrix games through a ranking value function method. *Journal of Mathematics and Computer Science-JMCS*, *18*(2), 175-183.

[4] Lemke, C. E., & Howson, Jr, J. T. (1964). Equilibrium points of bimatrix games. *Journal of the Society for Industrial and Applied Mathematics*, *12*(2), 413-423.

[5] Maeda, T. (2000). Characterization of the equilibrium strategy of the bimatrix game with fuzzy payoff. *Journal of Mathematical Analysis and Applications*, *251*(2), 885-896.

[6] Madandar, F., Haghayeghi, S., & Vaezpour, S. M. (2018). Characterization of Nash Equilibrium Strategy for Heptagonal Fuzzy Games. *International Journal of Analysis and Applications*, *16*(3), 353-367.

[7] Nayak, P. K., & Pal, M. (2009). The bi-matrix games with interval payoffs and its Nash equilibrium strategy. *Journal of Fuzzy Mathematics*, *17*(2), 421-435.

[8] Von Neumann, J., & Morgenstern, O. (2007). *Theory of games and economic behavior (commemorative edition)*. Princeton university press.

[9] Nash, J. (1951). Non-cooperative games. *Annals of mathematics*, 286-295.

[10] Owen. G. (1995). *Game Theory*. Third Edition Academic Press.

[11] Ryan, P., Egene, N., & yoavshoham. *Simple search methods for finding a Nash Equibrium*.standford university.

[12] Sakawa, M., & Nishizaki, I. (1997). Equilibrium solution in bi-matrix games with fuzzy pay-offs. *Japanese Fuzzy Theory and Systems*, *9*(3), 307-324.

[13] Cheng, S. F., Reeves, D. M., Vorobeychik, Y., & Wellman, M. P. (2004). Notes on equilibria in symmetric games.

[14] Kontogiannis, S., & Spirakis, P. (2011). Approximability of symmetric bimatrix games and related experiments. In *International Symposium on Experimental Algorithms*, 1-20.

[15] Sibasis, B., Prasun, K.N., & Madhumangal, P. (2013). Nash equilibrium solution in trapezoidal environment. *Journal of engineering*, 07-14.

[16] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.

[17] Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems*, *1*(1), 3-28.

[18] Gong, Z., & Hai, S. (2014). The interval-valued trapezoidal approximation of interval-valued fuzzy numbers and its application in fuzzy risk analysis. *Journal of Applied Mathematics*, *2014*.