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Domination Number of the Rough Zero Divisor Graph of the Rough Semiring

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Abstract: In this paper, we consider an approximation space $\mathrm{I} = (\mathrm{U}, \mathrm{R})$.where U is non empty finite set and R is an arbitrary equivalence relation on U. We define the dominating set of the Rough zero divisor graph $\mathrm{G}(\mathrm{Z}(\mathrm{T}^*))$ of the Rough Semiring T, where $\mathrm{Z}(\mathrm{T}^*)$ denotes the set of nonzero zero divisor of T. We construct a minimal dominating set on T. We also prove that the domination number of $\mathrm{G}(\mathrm{Z}(\mathrm{T}^*))$ is equal to the number of equivalence classes induced by R on U. We illustrate these concepts with suitable examples.

Keywords: Domination number, Dominating set, Rough Semiring, Semiring, Zero divisor, Zero divisor graph.

INTRODUCTION

Z. Pawlak [10] introduced the concept of rough set in 1982, a formal tool to process the incomplete information in the information system and it is defined as a pair of sets called lower and upper approximation. Hong et al. [7] dealt with some resultants over commutative idempotent Semiring in 2017. Zadeh [16] introduced the concept of fuzzy set theory in 1965 which is completely new approach to deal with vagueness and it is defined by membership in contrast to crisp membership. In 2013, Praba et al. [11] [12] [13] [14] gave a lattice structure on the set of all Rough sets on a given information system $I=(U,A)$. The least upper bound and greatest lower bound were established using the operations *Praba* Δ and *Praba* ∇. The author proved that T is not a Boolean algebra and derived the necessary condition for the existence of the complements of T. Hence (T,Δ,∇) is a Semiring called a Rough Semiring. In [8] [9] the authors detailed study of the riugh semiring and its homomorphism were established . S. Akbari et al[1] [2] discussed The Zero Divisor Graph of a Commutative Ring. David Dolzan and Polana Oblak [3] studied the concept of The Zero Divisor Graphs of Rings and Semirings.

In this paper, in section 2 we give preliminary definition that are require to study the forth coming section. In section 3, we prove that the cardinality of the minimal dominating set is equal to the number of equivalence classes induced by R on U and finally, we illustrate these concepts through examples in section 4.

Preliminaries

In this section we give preliminary definition related to rough set theory and graph theory

Rough Set Theory

Let U be a non empty finite set and R be an arbitrary equivalence relation on U then $I = (U,R)$ is called an approximation space and for $x \in U$, $[x]_R = \{ y \in U/ (x,y) \in R \}$ is said to be an equivalence class. Then, for $X \subseteq U$, let $RS(X)=\left(\underline{R}(X),\overline{R}(X)\right)$ be the rough set, where $\underline{R}(X)=\{x\in U \mid [x]_R \subseteq X \}$ is said to be a lower approximation and the upper approximation is defined as $\overline{R}(X)=\{x\in U \mid [x]_R \cap X \neq \phi\}$. Also the set of rough sets is defined as

$T = \{RS(X) | X \subseteq U\}$.

Definition 2.1.1[13]

Let X, Y ⊆ U. The Praba Δ is defined as

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XΔY = X ∪ Y, if IW(X ∪ Y) = IW(X) + IW(Y) − IW(X ∩ Y).

If IW(X ∪ Y) > IW(X) + IW(Y) − IW(X ∩ Y), then identify the equivalence class obtained by the union of X and Y. Then delete the elements of that class belonging to Y. Call the new set as Y. Now, obtain XΔY. Repeat this process until IW(X ∪ Y) = IW(X) + IW(Y) − IW(X ∩ Y). Where IW(X) is the number of equivalence classes contained in X.

Definition 2.1.2[13]

If X, Y ⊆ U then an element x ∈ U is called a Pivot element, if [x]p ⊈ X ∩ Y, but [x]p ∩ X ≠ φ and [x]p ∩ Y ≠ φ. The set of Pivot elements of X and Y is called the Pivot set of X and Y and it is denoted by $P_{X \cap Y}$

Definition 2.1.3[13]

Praba ∇ of X and Y is denoted by X∇Y and it is defined as X∇Y = {x | [x]p ⊆ X ∩ Y} ∪ PX∩Y where X, Y ⊆ U. Note that each Pivot element in $P_{X \cap Y}$ is the representative of that particular class.

Theorem 2.1.1[13]

Let I = (U, A) be an information system where U be the universal (finite) set and A be the set of attributes and T be the set of all rough sets then (T, Δ, ∇) is a Semiring.

Definition 2.1.4[14]

A subset X of U is said to be dominant if $X \cap X_i \neq \emptyset \, for \; i = 1,2, \dots n$.

Definition 2.1.5 [14] (Rough zero divisor)

Let (T, Δ, ∇) be a commutative rough semiring. An element $RS(X) \neq RS(\emptyset)$ of T is said to be a zero divisor of T if there exist $RS(Y) \neq RS(\emptyset)$ in T such that $RS(X)\nabla RS(Y) = RS(\emptyset)$ *i.e.,* $RS(X \nabla Y) = RS(\emptyset)$

Graph Theory

Definition 2.2.1[4]

A set D of vertices in a graph *G(V,E)* is said to be dominating set of G if every vertex not in D is adjacent to at least one vertex in D.

Definition 2.2.2[4]

A dominating set S is a minimal dominating set if no proper subset of S is a dominating set. The cardinality of smallest minimal dominating set of G is called the domination number and it is denoted by γ (G).

DOMINATION NUMBER OF ROUGH ZERO DIVISOR GRAPH

We know that a set $P \subseteq Z(T^*)$ is said to be a dominating set of the rough zero divisor graph $G(Z(T^*))$ if its satisfies the domination property namely if every element of $Z(T^*) - P$ is adjacent to at least one element in P and P is said to be a minimal dominating set if the removal of any element in P will affect the domination property. The number of elements in the smallest minimal dominating set is called the domination number of $G(Z(T^*))$.

In this section we consider an approximation space $I = (U, R)$ and let $\{X_1, X_2, \dots X_n\}$ are the equivalence classes induced by R on U.

We also assume that there are m equivalence classes $\{X_1, X_2, \dots X_m\}$ with cardinality greater than 1 and the remaining $n - m$ equivalence classes $\{X_{m+1}, X_{m+2}, \dots X_n\}$ have cardinality equal to 1, where $m \leq n$. Let $\{x_1, x_2, \dots x_m\}$ are the pivot elements (representative elements) of the equivalence classes with cardinality greater than 1.

Let $T = \{RS(X) | X \subseteq U\}$ be the Semiring with respect to the two operation $praba\Delta \; and \; praba\nabla$. Let $D = \{RS(x_1), RS(x_2), \dots RS(x_m), RS(X_{m+1}), RS(X_{m+2}) \dots RS(X_n)\}$.

Also we prove that D is the smallest minimal dominating set thereby proving that the domination number of the rough zero divisor graph of the rough semiring is equal to the number of equivalence classes induced by R on U.

Let $B_i = \{x_i, X_i, \emptyset\}$ for $i = 1,2, \dots n$ (Note that $B_{m+1} = (X_{m+1}, \emptyset)$, $B_{m+2} = \{X_{m+2}, \emptyset\} \dots B_n = \{X_n, \emptyset\}$)

Theorem

The set D is minimal dominating set for the rough zero divisor graph $G(Z(T^*))$ of the rough Semiring T.

Proof

First let us prove that D is the dominating set. Choose an element $RS(Y) \in Z(T^*)$. If Y contains atleast one pivot element then $RS(Y)$ is connected to $(X_{m+1}), RS(X_{m+2}), \dots RS(X_n)$. If Y does not contain any pivot element then $RS(Y)$ is connected to any one of $(x_1), RS(x_2), \dots RS(x_m)$. Therefore D is the Dominating set.

Next let us prove that D is minimal. We know that removal of any one element in D will affect the domination property. If we remove $RS(x_1)$ from D then let $D_1 = D - RS(x_1)$ then $RS(x_2 \cup x_3 \cup \dots \cup x_m \cup X_{m+1} \dots \cup X_n) \in Z(T^*)$ and it is not connected to any element in D_1. Hence D_1 cannot be a dominating set. Similar argument is true if we remove any of $RS(x_2), RS(x_3) \dots RS(x_m)$.

Now if $RS(X_{m+1})$ is removed from D and $D_2 = D - RS(X_{m+1})$ then $RS(x_1 \cup x_2 \cup \dots \cup x_m \cup X_{m+2} \dots \cup X_n) \in Z(T^*)$ and it is not connected to any element in D_2. Hence D_2 cannot be a dominating set. Similar argument is true if we remove any of $RS(X_{m+2}), RS(X_{m+3}) \dots RS(X_n)$. Therefore D is the minimal dominating set.

Theorem

The dominating number of the rough zero divisor graph $G(Z(T^*))$ is equal to the number of equivalence classes induced by R on U.

Proof

Already we have proved that $D = RS(x_1), RS(x_2), \dots RS(x_m), RS(X_{m+1}), RS(X_{m+2}) \dots RS(X_n)$ is a minimal Dominating set. Now we have to prove that D is the smallest minimal dominating set. That is to prove that if there exist another dominating set $\overline{D}$ of n_1 elements where $n_1 < n$ then $\overline{D}$ cannot be a minimal dominating set.

On the contrary, if we assume that $\overline{D}$ is a minimal dominating set consisting of n_1 elements where $n_1 < n$. Let $\overline{D} = \{ RS(Y_1), RS(Y_2), \dots RS\big((Y_{n_1}\big) \}$. As there are n equivalence classes and $\overline{D}$ contains n_1 elements where $n_1 < n$, elements of $\overline{D}$ cannot cover the representative of all the equivalence classes. Hence there exist atleast one equivalence class X_i such that either $RS(X_i) \cup A$ or $RS(x_i) \cup A$ does not belong to $\overline{D}$. Where A is the union of none, one or more of equivalence classes or their representative elements other than X_i.

Now A cannot contain the representative of all the remaining $n - 1$ equivalence classes (i.e excluding X_i) otherwise $X_i \cup A$ becomes a dominant set. Hence there exist atleast one equivalence classes say X_j or its representative element x_j. But in that case $x_1 \cup x_2 \cup \dots . x_{j-1} \cup x_{j+1} \cup \dots x_m \cup X_{m+1} \cup \dots X_n$ is not connected to any element in $\overline{D}$. But it belongs to $Z(T^*)$ .Hence $\overline{D}$ cannot be a dominating set which is a contradiction. Therefore D is the smallest minimal dominating set. Hence the domination number of $G(Z(T^*))$ is equal to n which is a number of equivalence classes induced by R on U.

Lemma

The vertex $RS(x_i)$ in $G(Z(T^*))$ is connected to $\alpha_i = \{RS(Y): Y = (Z_1 \cup Z_2 \dots Z_{i-1} \cup Z_{i+1} \dots Z_n\} \neq \emptyset$ where $Z_i \in B_i$ and for $i = 1,2, \dots n$.

Proof

Note that elements of α_i are of the form $RS(Y)$ where Y does not contain X_i or its pivot element x_i. Hence $RS(x_i) \nabla RS(Y) = RS(\emptyset)$. Hence $RS(x_i)$ and $RS(Y)$ are adjacent in $G\big(Z(T^*)\big)$.

Lemma

The vertex $RS(X_i)$ in $G(Z(T^*))$ is connected to $\beta_i = \{RS(Y): Y = (Z_1 \cup Z_2 \dots \cup Z_m \cup Z_{m+1 \dots} Z_{i-1} \cup Z_{i+1} \dots Z_n\} \neq \emptyset$ where $Z_i \in B_i$ and for $i = m+1, m+2, \dots n$.

Proof

Can be proved in similar lines as in lemma 3.1

Lemma

(i) The degree of $RS(x_i)$ in $G\big(Z(T^*)\big)$ is $3^{m-1} 2^{n-m} - 1$ for $i = 1,2, \dots n$.

(ii) The degree of $RS(X_i)$ in $G\big(Z(T^*)\big)$ is $3^m 2^{n-m-1} - 1$ for $i = m+1, m+2, \dots n$

Proof

(i) First let us find degree of $RS(x_1)$ by lemma 3.1 $RS(x_1)$ is connected to $\alpha_1 = \{RS(Y): Y = (Z_2 \cup Z_3 \cup \dots \cup Z_n\} \neq \emptyset$. Note that each Z_i, $i = 1,2, \dots m$ have three possible elements. Hence

$Z_2 \cup Z_3 \cup \dots \cup Z_m$ can be taken in 3^{m-1} ways. Also each Z_i for $i = m+1, m+2, \dots n$ have two possible elements which can be taken from B_i for $i = m+1, m+2, \dots n$. Hence $Z_{m+1} \cup Z_{m+2} \cup \dots \cup Z_n$ will have 2^{n-m} possible elements. Hence $RS(Y)$ in α_1 could be any of $3^{m-1}2^{n-m} - 1$ elements excluding $RS(\emptyset)$. Hence $RS(x_1)$ is connected to α_1, where $|\alpha_1| = 3^{m-1}2^{n-m} - 1$. Hence $|\alpha_i| = 3^{m-1}2^{n-m} - 1$ where $i = 1,2, \dots m$.
Therefore degree of $RS(x_i)$, $i = 1,2, \dots m$ is $3^{m-1}2^{n-m} - 1$.

(ii) The proof of this can be given in similar lines as above.

ILLUSTRATION

In this section we illustrate the above concept through example.

Example

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

Let $\{X_1, X_2, X_3\}$ are the equivalence classes induced by an equivalence relation R on U. Where

$X_1 = \{x_1, x_3\}$, $X_2 = \{x_2, x_4, x_6\}$, $X_1 = \{x_5\}$

$T = \{RS(\emptyset), RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1 \cup x_2\}), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2) RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(U)\}$

$Z(T^*) = \{RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1 \cup x_2\}), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3)\}$

Then the corresponding rough zero divisor graph $G(Z(T^*))$

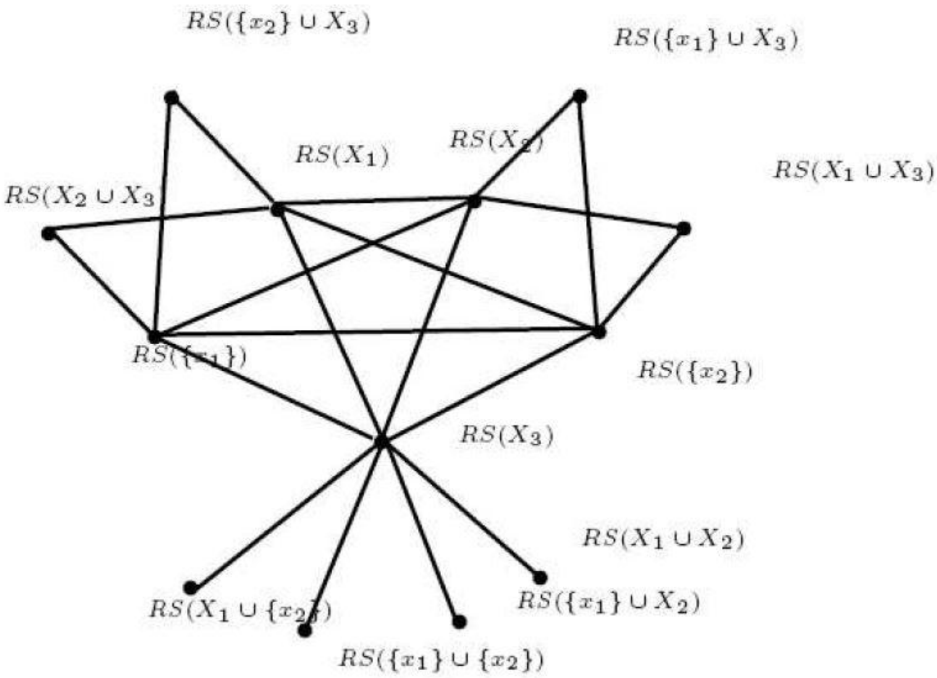

Figure1: Rough zero divisor graph

For this $G(Z(T^*))$, $D = \{RS(\{x_1\}), RS(\{x_2\}), RS(X_3)\}$ is the smallest minimal dominating set and the domination number is 3.

APPLICATION

In the following discussion we illustrate the use of rough zero divisor graph in medicine. Let U be the set of symptoms. Let X_1be the disease Malaria (D_1) with symptoms fever, sweating, vomiting, diarrhea and X_2 be the disease Heart attack (D_2) with symptoms abdominal pain, nausea, shortness of breath, fatigue. Clearly the set $\{X_1\}$ and $\{X_2\}$ forms a partition on U. Let $x_1 = fever$ and $x_2 = abdominal\ pain$ be the representative of the classes.

$T = \{RS(\emptyset), RS(X_1), RS(X_2), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1 \cup x_2\}, RS(\{x_1\} \cup X_2), RS(X_1 \cup \{x_2\}), RS(U)\}$

If $X = \{ sweating, nausea \}$ then $RS(X) = \{\emptyset, X_1 \cup X_2\}$ this means that these two symptoms together cannot confirm and D_1 and D_2 but these two are the symptoms of either D_1or D_2.

If $X = \{ fever, sweating, vomiting, diarrhea \}$then $RS(X) = \{ X_1, X_1\}$means that it conforms D_1. A similar interpretation can be given to all the rough sets in T.

The rough zero divisors $Z(T^*) = \{RS(X_1), RS(X_2), RS(\{x_1\}), RS(\{x_2\})\}$ and the rough zero divisor graph is

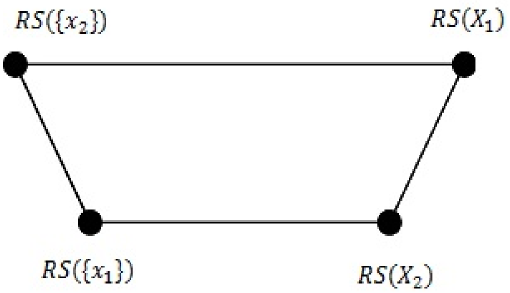

Figure 2: Rough zero divisor graph

Now for any two subsets X and Y of U. $X\nabla Y$ reflects the common symptoms with respect to the diseases D_1 and D_2. Hence if $RS(X)$ is a rough zero divisor then among the symptoms in U we are able to identify a subset Y that does not have any common symptom with X .in the rough zero divisor graph two such elements are connected. Therefore the edge $RS(X)$ to $RS(Y)$ in $G(Z(T^*))$ indicates that X and Y does not have any symptom in common. Also note that a Dominant set in this case represents a set of symptoms that exists with respect to both D_1 and D_2.

(i.e)., if $X = \{ fever, abdominal\ pain\}$ is a Dominant set and $RS(X) = (U, U)$ which means that it is necessary to check for both D_1 and D_2.

CONCLUSION

In this paper we have proved that D is a minimal dominating set of $G(Z(T^*))$ of rough semiring T. Also we have proved that the domination number of $G(Z(T^*))$ is equal to the number equivalence classes R on U.these concepts are illustrated through example. The future work in this direction is to explore the applications of domination number of rough zero divisor graph of rough semiring .

REFERENCES

[1] Akbari, S., & Mohammadian, A. (2004). On the zero-divisor graph of a commutative ring. *Journal of Algebra*, *274*(2), 847-855.

[2] Anderson, D. F., & Livingston, P. S. (1999). The zero-divisor graph of a commutative ring. *Journal of Algebra*, *217*(2), 434-447.

[3] Dolžan, D., & Oblak, P. (2012). The zero-divisor graphs of rings and semirings. *International Journal of Algebra and Computation*, *22*(04).

[4] Deo, N. (2017). *Graph theory with applications to engineering and computer science*. Courier Dover Publications.

[5] Ebrahimi Atani, S. (2008). The zero-divisor graph with respect to ideals of a commutative semiring. *Glasnik matematički*, *43*(2), 309-320.

[6] Ebrahimi Atani, S. (2009). An ideal based zero-divisor graph of a commutative semiring. *Glasnik matematički*, *44*(1), 141-153.

[7] Hong, H., Kim, Y., Scholten, G., & Sendra, J. R. (2017). Resultants over commutative idempotent semirings I: Algebraic aspect. *Journal of Symbolic Computation*, *79*, 285-308.

[8] Manimaran, A., Praba, B., & Chandrasekaran, V. M. (2014). Regular rough∇ monoid of idempotents. *International Journal of Applied Engineering and Research*, *9*(16), 3469-3479.

[9] Manimaran, A., Praba, B., & Chandrasekaran, V. M. (2017). Characterization of rough semiring. *Afrika Matematika*, *28*(5-6), 945-956.

[10] Pawlak, Z. (1982). Rough sets. *International journal of computer & information sciences*, *11*(5), 341-356.

[11] Praba, B., & Mohan, R. (2013). Rough lattice. *International Journal of Fuzzy Mathematics and System*, *3*(2), 135-151.

[12] Praba, B., Chandrasekaran, V. M., & Manimaran, A. (2013). A commutative regular monoid on rough sets. *Italian Journal of Pure and Applied Mathematics*, *31*, 307-318.

[13] Praba, B., Chandrasekaran, V. M., & Manimaran, A. (2015). Semiring on Rough sets. *Indian Journal of Science and Technology*, *8*(3), 280-286.

[14] Praba, B., Manimaran, A., & Chandrasekaran, V. M. (2015). The zero divisor graph of a rough semiring. *International Journal of Pure and Applied Mathematics*, *98*(5), 33-37.

[15] Visweswaran, S. (2011). Some properties of the complement of the zero-divisor graph of a commutative ring. *ISRN Algebra*, *2011*.

[16] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.